Kindergarten Standards

1. Representing and comparing whole numbers, initially with sets of objects
   - Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Describing shapes and space
   - Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Counting and Cardinality
Know number names and the count sequence.
K.CC.1: Count to 100 by ones and by tens.
K.CC.2: Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
K.CC.3: Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Counting to tell the number of objects.
K.CC.4: Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
K.CC.5: Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Comparing numbers.
K.CC.6: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Note: Include groups with up to ten objects.)
K.CC.7: Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking
Understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from.
K.OA.1: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Note: Drawings need not show details, but should show the mathematics in the problem – this applies wherever drawings are mentioned in the Standards.)
K.OA.2: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
K.OA.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
K.OA.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
K.OA.5: Fluently add and subtract within 5.

Number and Operations in Base Ten
Working with numbers 11 – 19 to gain foundations for place value.
K.NBT.1: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data
Describe and compare measurable attributes.
K.MD.1: Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
K.MD.2: Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Classify objects and count the number of objects in each category.
K.MD.3: Classify objects or people into given categories; count the numbers in each category and sort the categories by count. (Note: Limit category counts to be less than or equal to 10.)

Geometry
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
K.G.1: Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
K.G.2: Correctly name shapes regardless of their orientations or overall size.
K.G.3: Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional ("solid").

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
First Grade Standards

1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20
   - Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare strategies to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Developing understanding of whole number relationships and place value, including grouping in tens and ones
   - Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. The whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Developing understanding of linear measurement and measuring lengths as iterating length units
   - Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. (Note: students should apply the principle of transitivity of measurement to make direct comparisons, but they need not use this technical term.)

4. Reasoning about attributes of, and composing and decomposing geometric shapes
   - Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understandings of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.
1.OA.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (Note: See Glossary, Table 1.)

1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.
1.OA.3: Apply properties of operations as strategies to add and subtract. (Note: Students need not use formal terms for these properties.) Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

1.OA.4: Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

Add and subtract within 20.
1.OA.5: Relate counting to addition and subtraction (e.g., by counting on to add 2).
1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 = 12 + 1 = 13 + 1); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.
1.OA.7: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6; 7 = 5 + 2; 5 = 2 + 2; 4 = 1 + 5 = 2.
1.OA.8: Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + ? = 10.

Number and Operations in Base Ten

Extend the counting sequence.
1.NBT.1: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.
1.NBT.2: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
1.NBT.3: Compare two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. Use place value understanding and properties of operations to add and subtract.
1.NBT.4: Add within 100, including adding a two-digit number and a one-digit number, and adding two-digit numbers and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
1.NBT.5: Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count, explain the reasoning used.
1.NBT.6: Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data

Measure lengths indirectly and by iterating length units.
1.MD.1: Order three objects by length; compare the lengths of two objects indirectly by using a third object.
1.MD.2: Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit contexts to where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Tell and write time.
1.MD.3: Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.
1.MD.4: Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry

Reason with shapes and their attributes.
1.G.1: Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
1.G.2: Compose two-dimensional shapes (rectangles, squares, trapezoids, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Note: Students do not need to learn formal names such as "right rectangular prism.")
1.G.3: Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of, the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Second Grade Standards

1. Extending understanding of base-ten notation
   - Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. Building fluency with addition and subtraction
   - Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. Using standard units of measure
   - Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves iteration of units. They recognize that the shorter the unit, the more iterations they need to cover a given length.

4. Describing and analyzing shapes
   - Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding attributes of two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

Use place value understanding and properties of operations to add and subtract.

- 2.NBT.5: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

- 2.NBT.6: Add up to four two-digit numbers using strategies based on place value and properties of operations.

- 2.NBT.7: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

- 2.NBT.8: Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

- 2.NBT.9: Explain why addition and subtraction strategies work, using place value and the properties of operations. (Note: Explanations may be supported by drawings or objects.)

Measurement and Data

Measure and estimate lengths in standard units.

- 2.MD.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

- 2.MD.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

- 2.MD.3: Estimate lengths using units of inches, feet, centimeters, and meters.

- 2.MD.4: Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

- 2.MD.5: Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

- 2.MD.6: Represent whole-number sums and differences as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

- 2.MD.7: Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

- 2.MD.8: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Represent and interpret data.

- 2.MD.9: Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

- 2.MD.10: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph. (Note: See Glossary, Table 1.)

Geometry

Reason with shapes and their attributes.

- 2.G.1: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Note: Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

- 2.G.2: Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

- 2.G.3: Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, fourths, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
Operations and Algebraic Thinking

1. Developing understanding of multiplication and division and strategies for multiplication and division within 100

- Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is in finding the product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the number of objects in groups or the group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit numbers. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1)

- Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use unit fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Developing understanding of the structure of rectangular arrays and of area

- Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. The students connect tiling rectangles with whole number products to the area of rectangles. Students describe, analyze, and compare two-dimensional shapes. They reason inductively about attributes of shapes, making and testing conjectures about their properties. Students identify whether the rectangles can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect tiling rectangles with whole number products and justify the conclusion, e.g., by using a visual fraction model.

4. Describing and analyzing two-dimensional shapes

Students describe, analyze, and compare twodimensional shapes. They reason inductively about attributes of shapes, making and testing conjectures about their properties. Students identify whether the rectangles can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect tiling rectangles with whole number products and justify the conclusion, e.g., by using a visual fraction model.

Operations and Algebraic Thinking

3.OA.1: Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

3.OA.2: Interpret whole-as-a-whole quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

3.OA.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and area models; multiplication is in finding the product, and division is in finding an unknown factor in these situations. For equal-sized group situations, division can require finding the number of objects in groups or the group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit numbers. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

3.OA.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × ? = 72.

3.OA.5: Apply properties of operations as strategies to multiply and divide.

- Note: Students need not use formal terms for these properties. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as (8 × 5) + (8 × 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

3.OA.6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

3.OA.7: Multiply and divide within 100.

- Students describe, analyze, and compare twodimensional shapes. They reason inductively about attributes of shapes, making and testing conjectures about their properties. Students identify whether the rectangles can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect tiling rectangles with whole number products and justify the conclusion, e.g., by using a visual fraction model.

3.OA.8: Solve two-step word problems using the four operations. Represent these problems using number sentences with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Note: This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to modify a particular order—Order of Operations.)

3.OA.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

3.NF.1: Understand a fraction 1/b as a number that forms the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as a number formed by parts of size a formed by parts of size 1/b.

3.NF.2: Understand a fraction as a number on a number line. Represent fractions on a number line diagram.

- Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number line b on the number line.

3.NF.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- Understand fractions as equivalent if they represent the same point on a number line diagram. Recognize and generate simple equivalent fractions, e.g., 1/2 = 4/8, 5/10 = 1/2. Explain why the fractions are equivalent, e.g., by using visual fraction model.

- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3/1; recognize that 6/1 = 6 = locate 6/4 and 1 at the same point of a number line diagram.

- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1: Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Note: Excludes compound measures like)

3.MD.3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which a number of scissor pairs falls in each of three categories: red, green, and blue. Answer questions like “How many more red pairs than blue?” (Note: A range of algorithms may be used.)

3.MD.4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot with a scale of 1/2 inch per unit. (Note: A range of algorithms may be used.)

Geometric measurement: understand concepts of area and relate area to multiplication and addition.

3.MD.5: Recognize area as an attribute of plane figures and understand concepts of area measurement.

- A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.

- A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6: Measure areas by counting unit squares (square cm, square in, square ft, and improvised units).

3.MD.7: Relate area to the operations of multiplication and addition.

- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

- Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular area measures.

- Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.

- Recognize perimeters of figures as the sum of the lengths of the sides, and classify figures based on their perimeters.

Geometric measurement: recognize perimeter as an attribute of plane figures and understand concepts of perimeter.

3.G.1: Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
Fourth Grade – Standards

1. Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit divisors
   - Students generalize their understanding of place value to 1,000,000, understanding the sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers, and understand why these procedures work. They understand that models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate or mentally calculate quotients.

2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, multiplication of fractions by whole numbers
   - Students develop understanding of fraction equivalence and addition and subtraction of fractions with like denominators, multiplication of fractions by whole numbers based on the meaning of fractions, and the properties of operations. They recognize that a whole number is a multiple of each of its factors. Recognize that a fraction is equivalent to a fraction a/b is equivalent to a fraction n × a/b, for any whole number n (n ≠ 0).

3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry
   - Students generalize their understanding of perpendicular sides, particular angle measures, and symmetry of geometric figures. They understand that geometric figures can be analyzed and classified based on their properties such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Understand that geometric figures can be analyzed and classified based on their properties such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

4. Operations and Algebraic Thinking
   - Use the four operations with whole numbers to solve problems.

4.0A.3: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which drawings or algebraic expressions are used. Use analysis of drawings or algebraic expressions as strategies for solving problems. Gain familiarity with factors and multiples.

4.0A.4: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. Generate and analyze patterns.

4.0A.5: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate the terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate.

4.0B: Number and Operation in Base Ten
   - Number and Operations in Base Ten

4.B.1: Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.B.2: Understand the place value system.

4.0C.3: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.0D: Numbers and Operations – Fractions
   - Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 16, 20, 24, 40, and 100.

4.D.1: Interpreting a multiplication equation as a comparison.

4.D.2: Comparing two fractions with the same numerator or the same denominator.


Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Number and Operations – Fractions

5.NF.1: Recognize that a multiplier, a digit in one place represents 10 times as much as it represents in the place to its right, and that it represents in the plane to be left.

5.NF.2: Explain patterns in the number of zeros when multiplying a number by 10, and explain the placement of the decimal point when the product is multiplied by a power of 10. Use whole-number exponents to denote powers of 10.

5.NF.3: Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NF.4: Use place value understanding to round decimals to any place. Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NF.5: Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NF.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, properties of operations, and the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NF.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF.3: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4: Apply and extend previous understandings of multiplication to multiply a fraction by a fraction. a. Interpret the product (a/b) x q as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q x 1/b. For example, use a visual fraction model to show (2/3) x 4/5 = 8/15. (In general, a x (b/c) = a/b x c).

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show how the area can be decomposed into parts equal to the unit fraction multiplied by a whole number. Apply and extend previous understandings of division to divide unit fractions by non-zero whole numbers and whole numbers by unit fractions. (Note: Students should multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between division and multiplication. But division of a fraction by a fraction is not a requirement at this grade.)

i. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.

ii. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 ÷ (1/5) = 100.

iii. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. For example, by using visual fraction models or equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Measurement and Data

5.MD.1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

5.MD.2: Make a line plot to display a data set of measurements in fractions of a unit. 1/2. Represent actual data using unit fractions, with an understanding of how to record measurements in this grade to solve problems involving information presented in line plots. For example, given different measurements of Liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers was redistributed equally.

5.MD.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5: Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

5.MD.6: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the height by the area of the base. (Note: Students should multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between division and multiplication. But division of a fraction by a fraction is not a requirement at this grade.)

5.MD.7: Apply and extend previous understandings of division to divide unit fractions by other whole numbers and whole numbers by unit fractions. (Note: Students should multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between division and multiplication. But division of a fraction by a fraction is not a requirement at this grade.)

5.MD.8: Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to correspond to (0, 0). Give a point in the plane by an ordered pair of numbers, (x, y), and use the coordinate system to find distances between points with the same first coordinate or the same second coordinate.

5.G.2: Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Geometry.

Classify two-dimensional figures into categories based on their properties.

5.G.3: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5.G.4: Classify two-dimensional figures in a hierarchy based on properties.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fifth Grade – Standards

1. Developing fluency with addition and subtraction of fractions, developing understanding of the operations of multiplication of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

- Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent fractions with like denominators. They develop fluency in summing and subtracting. They develop fluency in summing and subtracting fractions and whole numbers.

2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.

- Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results.

3. Developing understanding of volume.

- Students recognize volume as an attribute of three-dimensional space. They understand that volume can be quantified by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1 by 1 by 1 unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems involving volume. They decompose three-dimensional shapes into arrays of cubes and measure necessary attributes of shapes in order to solve real-world and mathematical problems.

Operations and Algebraic Thinking

Write and interpret numerical expressions.

5.OA.1: Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as (8 + 7) × 2. Recognize that 3 × (182/9 + 291) is three times as large than 182/9 + 291, without having to calculate the indicated sum or product.

Analyze patterns and relationships.

5.OA.3: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form and graph the ordered pairs on a coordinate plane. Explain informally why this is so.
Sixth Grade Standards

## Writing, interpreting, and using expressions and equations

- **Standard 1:** Write and interpret numerical expressions, and write, read, and evaluate expressions in which letters stand for numbers.
  - **6.EE.1:** Write and evaluate numerical expressions involving whole-number exponents.
  - **6.EE.2:** Write, read, and evaluate expressions in which letters stand for numbers.
    - **6.EE.2.a:** Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as 5 - y.
    - **6.EE.2.b:** Identify parts of an expression using mathematical terms (sum, term, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (2 + 7) as a product of two factors; view (3 + 8) as a single entity and 3 (2 + 8) as a sum of two terms.
    - **6.EE.2.c:** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the distributive property to express the area of a rectangle with side lengths 92 feet by 15 feet as 92 * 15 as a sum of two products; view (3 * 5) * 2 as a product of three factors.

## Ratios and Proportional Relationships

- **Standard 1:** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.". For every vote candidate A received, candidate C received half as many votes. For every lap that candidate A completed, candidate C completed two laps.

- **Standard 2:** Use ratio and rate reasoning to solve real-world and mathematical problems. For example, by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  - **6.RP.1:** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." For every vote candidate A received, candidate C received half as many votes. For every lap that candidate A completed, candidate C completed two laps.
  - **6.RP.2:** Use ratio and rate reasoning to solve real-world and mathematical problems. By reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    - **6.RP.2.a:** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
    - **6.RP.2.b:** Unit rate problems including those involving unit pricing and constant speed. Use tables with related rates or with equivalent ratios to solve problems. For example, if a recipe calls for 2 cups of flour to make 12 cookies, how much flour would be needed to make 36 cookies?
    - **6.RP.2.c:** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of $-30 dollars, write |$-30$| to describe the size of the debt in dollars.
    - **6.RP.2.d:** Distances, in particular negative of the number line, and coordinates of the points are on the number line.

## The Number System

- **Standard 1:** Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
  - **6.NS.1:** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions. Use visual fraction models or equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 2/3 ÷ 3/4 = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2 1/2 cups of yogurt? How wide is a rectangular strip of land with length 4/5 miles and width 2/3 mile?

- **Standard 2:** Compute fluently with multi-digit numbers and find common factors and multiples.
  - **6.NS.2:** Fluently divide multi-digit numbers using the standard algorithm.

- **Standard 3:** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

- **Standard 4:** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

- **Standard 5:** Apply and extend previous understandings of numbers to the system of rational numbers. For example, express 3/4 as 6/8 and 1.5/4 as 3/8; write 7/12 as 28/96.
  - **6.NS.3:** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
  - **6.NS.4:** Find the Greatest Common Factor of Two Whole Numbers. For example, find the greatest common factor of 120 and 130.

- **Standard 6:** Apply and extend previous understandings of numbers to the system of rational numbers. For example, express 3/4 as 6/8 and 1.5/4 as 3/8; write 7/12 as 28/96.
  - **6.NS.5:** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, altitude above/below sea level, credit/debit, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
  - **6.NS.6:** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
    - **6.NS.6.a:** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.
    - **6.NS.6.b:** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections of each other across one or both axes.
    - **6.NS.6.c:** Find and position integers and other rational numbers on a number line diagram.

- **Standard 7:** Apply and extend previous understandings of numbers to the system of rational numbers. For example, express 3/4 as 6/8 and 1.5/4 as 3/8; write 7/12 as 28/96.
  - **6.NS.7:** Understand ordering and absolute value of rational numbers.
    - **6.NS.7.a:** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
    - **6.NS.7.b:** Write, read, and compare rational numbers in the form of fractions and decimals.
    - **6.NS.7.c:** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -$30 dollars, write |$-30$| to describe the size of the debt in dollars.
    - **6.NS.7.d:** Distances, in particular negative of the number line, and coordinates of the points are on the number line.

## Expressions and Equations

- **Standard 1:** Apply and extend previous understandings of arithmetic to algebraic expressions.
  - **6.EE.5:** Write and evaluate expressions in which字母s appear in more than one place.
  - **6.EE.6:** Use variables to represent two quantities in a real-world situation.
  - **6.EE.7:** Write inequalities of the form x > c or x < c.

## Geometry

- **Standard 1:** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.

- **Standard 2:** Represent three-dimensional figures using nets of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

## Statistics and Probability

- **Standard 1:** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

- **Standard 2:** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and shape.

- **Standard 3:** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variability can describe how its values vary with a single number.

## Mathematical Practices

- **1.** Make sense of problems and persevere in solving them.
- **2.** Reason abstractly and quantitatively.
- **3.** Construct viable arguments and critique the reasoning of others.
- **4.** Model with mathematics.
- **5.** Use appropriate tools strategically.
- **6.** Attend to precision.
- **7.** Look for and make use of structure.
- **8.** Look for and express regularity in repeated reasoning.
Seventh Grade Standards:

1. Developing understanding of and applying proportional relationships
   a. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems.
   b. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease.
   c. Solve problems involving scale factors and similarity.
   d. Understand and use ratios involving units of different dimensions.

2. Developing understanding of operations with rational numbers and working with expressions and linear equations
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
   b. Develop a probability model (which may not be uniform) by assigning probabilities to outcomes and using these probabilities to make predictions. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Use this information to predict outcomes. Note: The estimation of the frequency of a chance event is not the same as the probability that event will happen. One reason for this is that the estimation is not the same as the probability that event will happen. One reason for this is that the estimation is not the same as the probability that event will happen. One reason for this is that the estimation is not the same as the probability that event will happen.
   c. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
   b. Develop a probability model (which may not be uniform) by assigning probabilities to outcomes and using these probabilities to make predictions. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Use this information to predict outcomes. Note: The estimation of the frequency of a chance event is not the same as the probability that event will happen. One reason for this is that the estimation is not the same as the probability that event will happen. One reason for this is that the estimation is not the same as the probability that event will happen. One reason for this is that the estimation is not the same as the probability that event will happen.
   c. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

4. Drawing inferences about populations based on samples
   a. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences and similarities in data sets involving two populations using random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. For example, if it takes 4 hours to mow 1/4 acre, then the unit rate is 1/4 acre per hour.
   d. Explain what a proportionality constant means in terms of the situation, with special attention to the points (0, 0) and (1, k) where k is the unit rate.

7.RP.3: Use proportional relationships to solve multistep ratio and percent problems.
   a. Solve problems involving percentages of a number such as determining sales tax, tips, and percent increase or decrease.
   b. Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example, if a woman making $25 an hour gets a 10% raise, she will make an additional $1/10 of her salary an hour, or $2.50, for a new salary of $27.50.
   c. Solve multi-step real-world and mathematical problems involving the four operations with rational numbers. Use equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
   d. Solve problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example, as a salesperson, you are paid $50 plus $3 per call. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solution.

7.SP.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce unbiased results about the population.
   a. Draw informal comparative inferences about a population from random samples.
   b. Draw informal comparative inferences about a population from random samples.
   c. Draw informal comparative inferences about a population from random samples.
   d. Draw informal comparative inferences about a population from random samples.

7.SP.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or parallel simulations) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words in a book, predict the winner of a school election based on randomly sampled survey data, gauge how far off the estimate might be.

7.SP.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between two heights of players is equal to one mean absolute deviation.
The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1: Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate or eventually repeat. Know that other numbers are called irrational.

8.NS.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π²). For example, by truncating the decimal expansion of √2, show that 2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue or generalize the argument.

Work with radicals and integer exponents.

8.EE.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3² × 3⁻⁵ = 3⁻³ = 1/3³ = 1/27.

8.EE.2: Use square root and cube root symbols to represent solutions to equations of the form x² = p and x³ = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

8.EE.3: Use numbers expressed in the form of a single digit times a whole number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 × 10⁸ and the population of the world as 7 × 10⁹, and determine that the world population is more than 20 times larger.

8.EE.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph, and comparing two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.6: Use similar triangles to explain why the slope m of a line is the same between any two distinct points on the line, and use the equation y = mx + b to describe the graph of a linear equation.

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.7: Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8: Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates of two pairs of points intercepts the line through the second pair.

Statistics and Probability

Investigate patterns of association in bivariate data.

8.SP.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities.

a. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

b. Know that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to determine possible association between the two variables. For example, collect data from students in your class on an environmental issue and make a bar graph showing the results. Ask if there is a association between students' gender and their answer to the survey question.

8.SP.2: Know that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to determine possible association between the two variables. For example, collect data from students in your class on an environmental issue and make a bar graph showing the results. Ask if there is a association between students' gender and their answer to the survey question.

Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1: Verify experimentally the properties of rotations, reflections, and translations.

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

8.G.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, translations. Given two two-dimensional figures, describe a sequence that exhibits the congruence between them.

8.G.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two two-dimensional figures, describe a sequence that exhibits the similarity between them.

Mathematical Practices

1. Make sense of problems and persevere in solving them. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
**Critical Area 1**

By the end of grade 8, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of two linear equations. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations (e.g., f(x) = mx + b, y = ax² + bx + c, and y = (x − p)(x − q), etc.). They understand the process of solving equations as the mathematical process of answering the question: “What number from the domain of the equation is a solution to the equation?” They interpret solutions in the context of problems; they also prove that two equations are equivalent using algebra (i.e., by showing that they have the same solution set). Students reason about and solve simple one-variable inequalities. They write and interpret inequalities of the form x > c or x < c. They graph solutions of linear inequalities in two variables.

**Critical Area 2**

In earlier grades, students define, evaluate, and compare functions. In this unit, students learn function notation and develop the concepts of domain and range as they apply the equal sign and use substitution to evaluate functions for inputs in their domain. Students focus on creating quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

**Critical Area 3**

In this unit, students consider quadratic functions, which may have linear, quadratic, or exponential models. They relate and compare concepts to linear and exponential functions. Students extend their experience with quadratics to exponential functions. They solve problems involving exponential growth and decay.

## Unit 1: Linear and Exponential Relations

### Cluster 1: Linear and Exponential Functions

Students learn to use linear and exponential functions to describe, interpret, and compare two quantities, where one is linear in relation to the other.

**Lesson N.11:** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those exponents, allowing for a notation for radicals where appropriate in terms of rational numbers. For example, we define 5^(1/3) to be the n-th root of 5, where n is a positive integer. (For n = 2, 3, 4, 5, ..., n corresponds to the 2nd, 3rd, 4th, 5th, ... roots, respectively.) (N.RN.1)

### Cluster 2: Linear Equations and Inequalities

Students extend their understanding of solving equations to solving linear equations and inequalities in one variable and systems of equations.

**Lesson A.REI.7:** Solve a variety of systems of linear equations in two variables using multiple strategic approaches, and justify why these methods work. (A.REI.8)

### Cluster 3: Systems of Linear Equations

Students extend their experience with systems of linear equations to solving a variety of systems in two or more variables. They identify and interpret solutions. Students distinguish between situations where the system of equations has no solution, one solution, or infinitely many solutions. (A.REI.6)

**Lesson A.REI.10:** Understand that the graph of a function in two variables is all of its solutions plotted in the coordinate plane, often forming a curve or surface. (A.REI.11)

### Cluster 4: Linear Functions

Students extend their understanding of linear functions to include situations with variables as inputs and outputs of functions. They interpret solutions of linear equations and inequalities in terms of given contexts. (A.REI.2)

**Lesson A.REI.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A.REI.12)

### Cluster 5: Quadratic Functions

Students extend their understanding of quadratic functions. They interpret solutions of quadratic equations as the zeros of quadratic functions. They distinguish between situations where the quadratic model is appropriate and where another model is more appropriate. (A.REI.4)

**Lesson A.REI.4:** Explain why the solutions to a quadratic equation are the roots of one of the x-intercepts of the graph of a quadratic function. (A.REI.5)

### Cluster 6: Exponential Functions

Students extend their understanding of exponential functions. They interpret solutions of exponential equations as the intersections of exponential graphs with horizontal or vertical lines. (A.REI.6)

**Lesson A.REI.6:** Solve exponential equations analytically, graphically, and numerically (e.g., using technology to graph the functions, make tables of values, and find zeroes); compare results. (A.REI.7)

### Cluster 7: Functions

Students extend their understanding of functions to include situations with inputs and outputs of functions being a set of related or equivalent terms, such as in a table, graph, equation, or expression. They interpret solutions of functions in terms of given contexts. (A.REI.8)

**Lesson A.REI.8:** Represent and solve simple (i.e., first-degree) equations graphically. (A.REI.9)

### Cluster 8: Inequalities

Students extend their understanding of inequalities to include situations with inputs and outputs of functions being a set of related or equivalent terms, such as in a table, graph, equation, or expression. They interpret solutions of inequalities in terms of given contexts. (A.REI.10)

**Lesson A.REI.10:** Understand that the graph of an equation in two variables is all of its solutions plotted in the coordinate plane, often forming a curve or surface. (A.REI.11)

### Cluster 9: Systems of Inequalities

Students extend their understanding of systems of equations to systems of inequalities. They interpret solutions of systems of inequalities as regions in the coordinate plane. (A.REI.12)

**Lesson A.REI.12:** Graph the solutions to a system of linear inequalities in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A.REI.13)

### Cluster 10: Functions

Students extend their understanding of functions to include situations with inputs and outputs of functions being a set of related or equivalent terms, such as in a table, graph, equation, or expression. They interpret solutions of functions in terms of given contexts. (A.REI.8)

**Lesson A.REI.8:** Represent and solve simple (i.e., first-degree) equations graphically. (A.REI.9)

### Cluster 11: Exponential Functions

Students extend their understanding of exponential functions. They interpret solutions of exponential equations as the intersections of exponential graphs with horizontal or vertical lines. (A.REI.6)

**Lesson A.REI.6:** Solve exponential equations analytically, graphically, and numerically (e.g., using technology to graph the functions, make tables of values, and find zeroes); compare results. (A.REI.7)

### Cluster 12: Functions

Students extend their understanding of functions to include situations with inputs and outputs of functions being a set of related or equivalent terms, such as in a table, graph, equation, or expression. They interpret solutions of functions in terms of given contexts. (A.REI.8)

**Lesson A.REI.8:** Represent and solve simple (i.e., first-degree) equations graphically. (A.REI.9)

### Cluster 13: Inequalities

Students extend their understanding of inequalities to include situations with inputs and outputs of functions being a set of related or equivalent terms, such as in a table, graph, equation, or expression. They interpret solutions of inequalities in terms of given contexts. (A.REI.10)

**Lesson A.REI.10:** Understand that the graph of an equation in two variables is all of its solutions plotted in the coordinate plane, often forming a curve or surface. (A.REI.11)

### Cluster 14: Systems of Inequalities

Students extend their understanding of systems of equations to systems of inequalities. They interpret solutions of systems of inequalities as regions in the coordinate plane. (A.REI.12)

**Lesson A.REI.12:** Graph the solutions to a system of linear inequalities in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (A.REI.13)
Kentucky Core Academic Standards

Unit 3: Descriptive Statistics

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. In this unit, students apply their ability to see structure in expressions to reveal and create quadratic and exponential expressions. They solve and explain equations, inequivalences, and systems of equations involving exponents.

Clusters with Instructional Notes

Unit 4: Expressions and Equations

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students use their ability to see structure in expressions to reveal and create quadratic and exponential expressions. They solve and explain equations, inequivalences, and systems of equations involving exponents.

Clusters with Instructional Notes

Unit 5: Quadratic Functions and Modeling

In preparation for work with quadratic relationships, students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students use their ability to see structure in expressions to reveal and create quadratic and exponential expressions. They solve and explain equations, inequivalences, and systems of equations involving exponents.
Understanding congruence in terms of rigid motions. Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle, which are assumed without proof) rigid motions and their assumed properties can be used to establish the validity of the triangle congruence criteria, which can then be used to prove other theorems. Prove theorems about lines and angles.

Understand similarity in terms of similarity transformations. Similarity transformations include rotations, reflections, and parallel translations. Some of these transformations are used to determine intersections between lines and circles or parabolas and hence are important for the development of formal proofs. Students prove theorems using a variety of formats, including those that will carry a given figure onto another. Understand similarity in terms of similarity transformations.

Prove theorems about lines and angles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Understand similarity in terms of similarity transformations. Similarity transformations include rotations, reflections, and parallel translations. Some of these transformations are used to determine intersections between lines and circles or parabolas and hence are important for the development of formal proofs. Students prove theorems using a variety of formats, including those that will carry a given figure onto another. Understand similarity in terms of similarity transformations.

G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right-angled triangle problems. Problems include those that can be solved with the use of a calculator.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

G.MG.4 Derive the formula A = 1/2 ab sin C for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to an opposite side.

G.SRT.10 Prove the laws of Sines and Cosines and use them to solve problems.

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Unit 3: Extending to Three Dimension

Some of the two-dimensional and three-dimensional objects is extended to informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a given axis.

G.MG.1 Develop informal arguments for circumference of circles, area of circles, and surface area of prisms, pyramids, and cylinders.

G.MG.2 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

G.MG.4 Derive the formula A = 1/2 ab sin C for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to an opposite side.

G.SRT.10 Prove the laws of Sines and Cosines and use them to solve problems.

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Unit 4: Geometry and Algebra

Understand similarity in terms of similarity transformations. Similarity transformations include rotations, reflections, and parallel translations. Some of these transformations are used to determine intersections between lines and circles or parabolas and hence are important for the development of formal proofs. Students prove theorems using a variety of formats, including those that will carry a given figure onto another. Understand similarity in terms of similarity transformations.

Prove theorems about lines and angles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

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G.SRT.10 Prove the laws of Sines and Cosines and use them to solve problems.

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Defining theorems that are the relationships of triangles in the plane, leading to definitions of trigonometric ratios for acute angles.

G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right-angled triangle problems. Problems include those that can be solved with the use of a calculator.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

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G.SRT.10 Prove the laws of Sines and Cosines and use them to solve problems.

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Apply geometric concepts in modeling situations. Focus on situations well modeled by trigonometric ratios for acute angles.

Apply trigonometry to general triangles. With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.
Unit 4: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8th grade to find distances, students use the rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Unit 5: Circles With and Without Coordinates

In this unit, students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or parabolas and between two circles.

Unit 6: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability (e.g., probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
Unit 2: Trigonometric Functions

Building on their previous work with functions, and on their work with trigonometric ratios and circles in geometry, students now use the coordinate plane to extend trigonometric functions to model periodic phenomena.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Kentucky Core Academic Standards

<table>
<thead>
<tr>
<th>Cluster with Instructional Notes</th>
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<tbody>
<tr>
<td>F.TF.1</td>
<td>Extend domain of trigonometric functions using the unit circle.</td>
</tr>
<tr>
<td>F.TF.2</td>
<td>Extend the domain of trigonometric functions using the unit circle.</td>
</tr>
<tr>
<td>F.TF.3</td>
<td>Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.</td>
</tr>
<tr>
<td>F.TF.4</td>
<td>Prove and use trigonometric identities.</td>
</tr>
<tr>
<td>F.TF.5</td>
<td>Prove and use trigonometric identities.</td>
</tr>
<tr>
<td>F.TF.6</td>
<td>Prove the Pythagorean identity sin²θ + cos²θ = 1 and use it to find sin θ, cos θ, or tan θ, given sin θ, cos θ, or tan θ, and the quadrant of the angle.</td>
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<td>F.TF.1</td>
<td>Understand radian measure as the length of the arc on the unit circle subtended by the angle.</td>
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<tr>
<td>F.TF.2</td>
<td>Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</td>
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<td>F.TF.3</td>
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Model periodic phenomena with trigonometric functions. Prove and apply trigonometric identities. Extend the domain and addition subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could be limited to acute angles in Algebra II.
Unit 3: Modeling with Functions

In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as ‘the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions’ is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and probability is employed in modeling contexts.

Kentucky Core Academic Standards

Clusters with Instructional Notes

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R.

F.BF.4 A For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features (intercepts and end behavior, and maximum or minimum values). For example, f(x) = 2x3 or f(x) = (x+1)/(x-1) for specific values of k (both positive and negative); find the value of k given the graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.4 F For inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, (f(x) + 2)3 = 8, c = 8/3.

F.LE.4 F For exponential models, express as a logarithm the solution to f(x) = c for simple functions that have an inverse and write an expression for the inverse. For example, f(x) = cx or f(x) = √x+2 or f(x) = ln(x).

New functions from existing functions

Use transformations of functions to find models as students consider increasingly more complex situations.

For F.BF.3, note the effect of multiple transformations on a single graph and the common effect of each transformation across function types. Extend F.BF.4 to simple rational, simple radical, and simple exponential functions; connect F.BF.4 to F.LE.4. Construct and compare linear, quadratic, and exponential models and solve problems. Consider extending this unit to include the relationship between properties of logarithms and properties of exponents; such as the connection between the properties of exponents and the basic logarithm property that logb(xy) = logb x + logb y.

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Unit 4: Inferences and Conclusions from Data

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different types of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Summary, represent, and interpret data on a single count or measurement variable. While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students’ understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.

Understand and evaluate random processes underlying statistical experiments.

For S.I.C.2, include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

For S.I.C.4, use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

For S.I.C.5, use data from a randomized experiment to compare two treatments; use simulations to decide if differences between treatments are significant.

For S.I.C.6, Evaluate reports based on data.

F.LE.4 F For exponential models, express as a logarithm the solution to f(x) = c for simple functions that have an inverse and write an expression for the inverse. For example, f(x) = cx or f(x) = √x+2 or f(x) = ln(x).

Analyzing functions using different representations.

Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.

Build a function that models a relationship between two quantities.

Develop models for more complex or sophisticated situations than in previous courses.

F.BF.4 Identify the effect on the graph of replacing f(x) by f(x) + k, f(x) + k, (f(x) + k), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.4 F For inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, (f(x) + 2)3 = 8, c = 8/3.

F.LE.4 F For exponential models, express as a logarithm the solution to f(x) = c for simple functions that have an inverse and write an expression for the inverse. For example, f(x) = cx or f(x) = √x+2 or f(x) = ln(x).

New functions from existing functions

Use transformations of functions to find models as students consider increasingly more complex situations.

For F.BF.3, note the effect of multiple transformations on a single graph and the common effect of each transformation across function types. Extend F.BF.4 to simple rational, simple radical, and simple exponential functions; connect F.BF.4 to F.LE.4. Construct and compare linear, quadratic, and exponential models and solve problems. Consider extending this unit to include the relationship between properties of logarithms and properties of exponents; such as the connection between the properties of exponents and the basic logarithm property that logb(xy) = logb x + logb y.